Bass Diffusion Model

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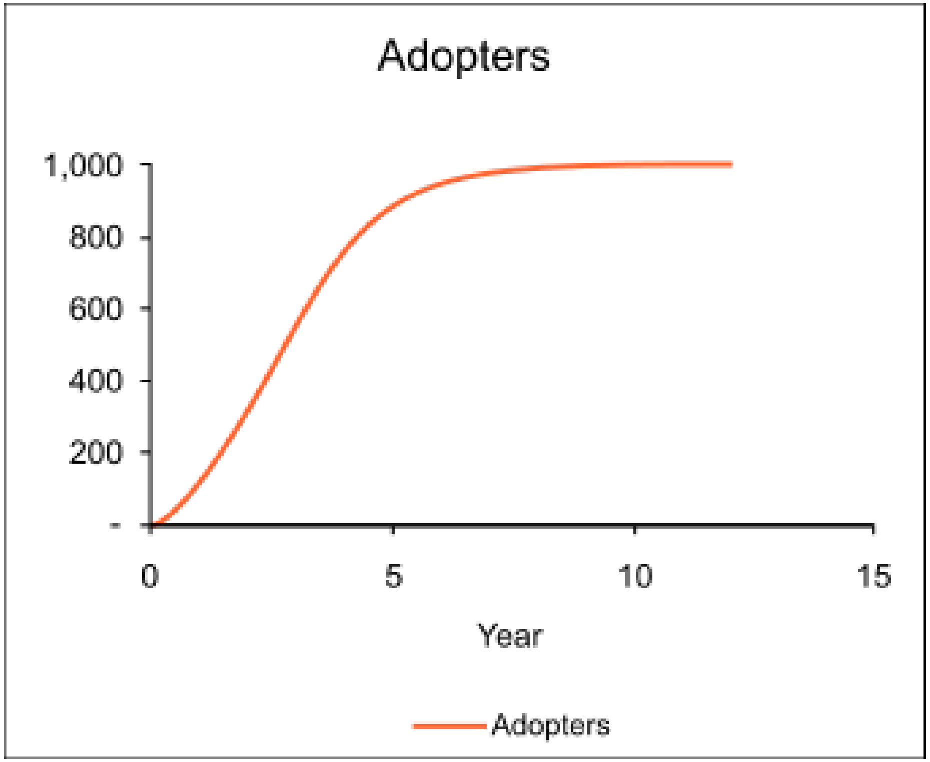
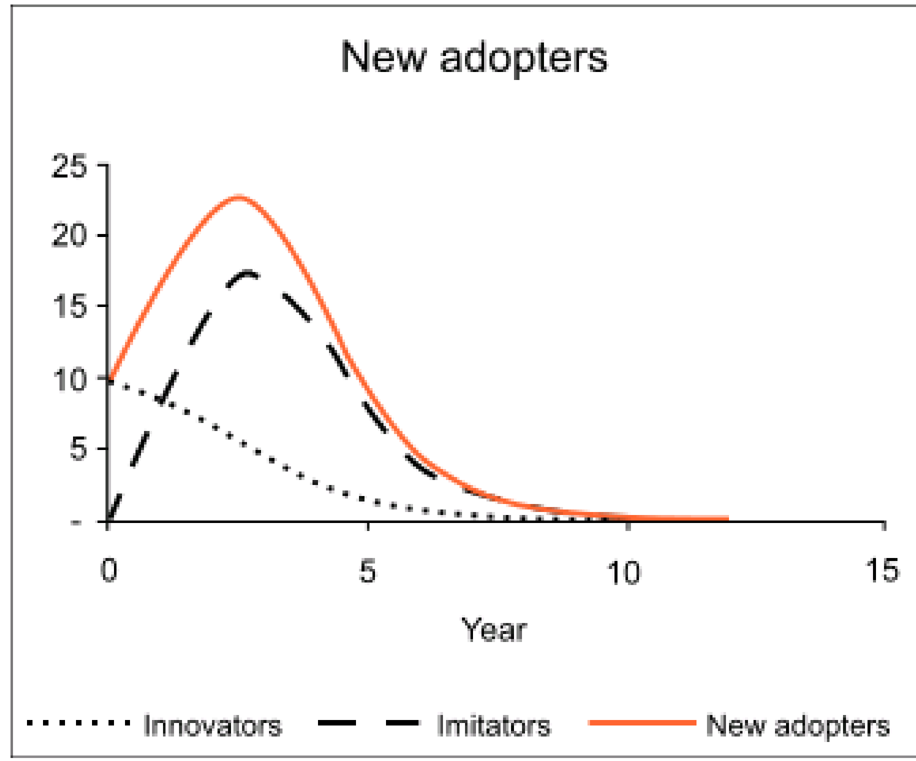
Innovation & Imitation





Frank M. Bass 1926 - 2006

- Professor of Marketing Science in the School of Management at the University of Texas at Dallas until his death in December 2006.
- During his 53 years as a professor, he published more than 100 papers and supervised 60 Ph.D. students.
- He made important contributions to econometrics, stochastic brand choice, and new-product diffusion modeling.
- His classic paper on the "Bass Model" was named by INFORMS as one of the Ten Most Influential Papers published in the 50-year history of *Management Science*.

Introduction

- The Bass Model parameter representing the potential market, which is the ultimate number of purchasers of the product, is constant. It is denoted by M.

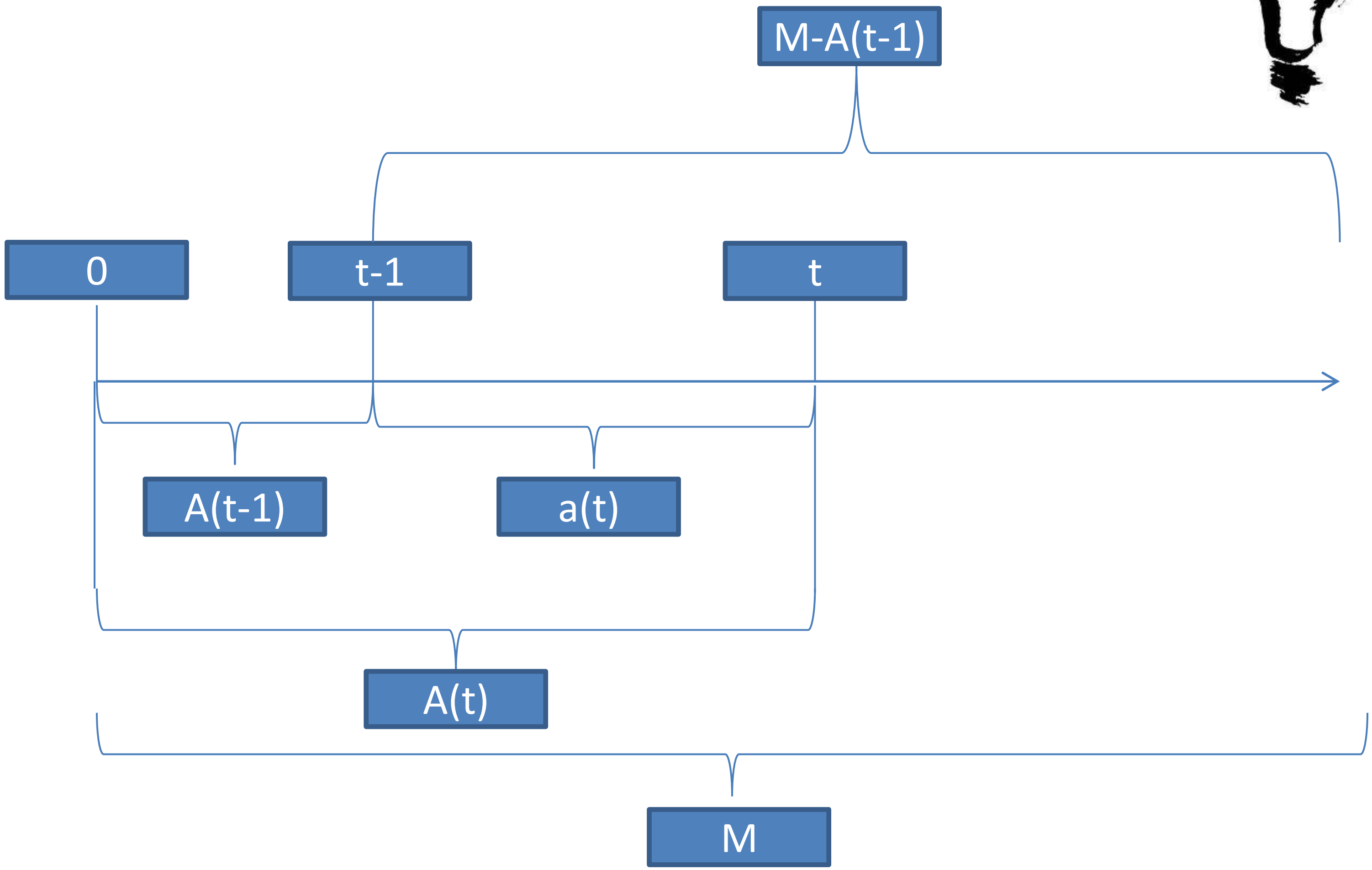
- Time intervals are numbered sequentially with the first full time interval (usually year) of sales at $t = 1$ in the Srinivasan-Mason[2] form of the Bass Model equations. A time interval is denoted t.

- The portion (fraction) of the potential market that adopts at time t is $f(t)$.
- The portion (fraction) of the potential market that has adopted up to and including time t is $F(t)$.

- The number of the potential market that adopts at time t is $a(t)$.
- The number of the potential market that has adopted up to and including time t is $A(t)$.

- The Bass model coefficient (parameter) of innovation is p.
- The Bass model coefficient (parameter) of imitation is q.

discrete-time model
a small time increment



$$A(t) = A(t-1) + p\big(M - A(t-1)\big) + q\,\frac{A(t-1)}{M}\big(M - A(t-1)\big)$$

Hazard rate

- The interpretation of the hazard is that if it is multiplied by a small time increment it gives the probability that a random purchaser who has not yet made the purchase will do so in the next small time increment.

- Hazard rate is "the portion that adopts at t given that they have not yet adopted"
- Conditional probability: $p(A|B) = \frac{A \cap B}{B}$

$$h(t) = \frac{f(t)}{1-F(t)} \qquad (1)$$

- "The probability of adopting by those who have not yet adopted is a linear function of those who had previously adopted."

$$h(t) = p + qF(t) \qquad (2)$$

$$\frac{f(t)}{1-F(t)} = p + qF(t)$$

$$\begin{cases} A(t) = M\,F(t) \text{ and } a(t) = M\,f(t) \\ a(t) = Mp + [q - p]\,A(t) - \dfrac{q}{M}A(t)^2 \end{cases}$$

$$f(t) = p + [q - p]\,F(t) - q\,[\,F(t)]^2$$

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \dfrac{q}{p}e^{-(p+q)t}} \longrightarrow f(t) = \frac{\dfrac{(p+q)^2}{p}e^{-(p+q)t}}{\left(1 + \dfrac{q}{p}e^{-(p+q)t}\right)^2}$$

Solving the differential equation of bass diffusion model

- http://www.bassbasement.org/F/N/FMB/Pubs/Bass%201969%20New%20Prod%20Growth%20Model.pdf

in order to find $F(T)$ we must solve this non-linear differential equation:

$$dT = dF/(p + (q - p)F - qF^2).$$

The solution is:

$$F = (q - pe^{-(T+C)(p+q)})/q(1 + e^{-(T+C)(p+q)}).$$

Since $F(0) = 0$, the integration constant may be evaluated:

$$-C = (1/(p + q)) \operatorname{Ln}(q/p) \quad \text{and} \quad F(T) = (1 - e^{-(p+q)T})/(q/pe^{-(p+q)T} + 1)$$

Then,

$$f(T) = ((p + q)^2/p)[e^{-(p+q)T}/(q/pe^{-(p+q)T} + 1)^2],$$

and

$$S(T) = (m(p + q)^2/p[e^{-(p+q)T}/(q/pe^{-(p+q)T} + 1)^2].$$

Using Mathematica

- Since F(0)=0
- $\mathrm{DSolve}[\{F'[t] == p + (q-p) * F[t] - q * (F[t])^2, F[0] == 0\}, \mathrm{F}, t]$
- $\left\{\left\{f \to \mathrm{Function}\left[\{t\}, \frac{e^{pt+qt} + \left(-\frac{1}{p}\right)^{\frac{p}{p+q}+\frac{q}{p+q}} p}{e^{pt+qt} - \left(-\frac{1}{p}\right)^{\frac{p}{p+q}+\frac{q}{p+q}} q}\right]\right\}\right\}$
- F(t)= $\frac{1-e^{-(p+q)t}}{1+\frac{\mathrm{q}}{p}e^{-(p+q)t}}$

Relationship with other s-curves

- There are two special cases of the Bass diffusion model.

1. The first special case occurs when q=0, when the model reduces to the Exponential distribution.
2. The second special case reduces to the logistic distribution, when p=0.

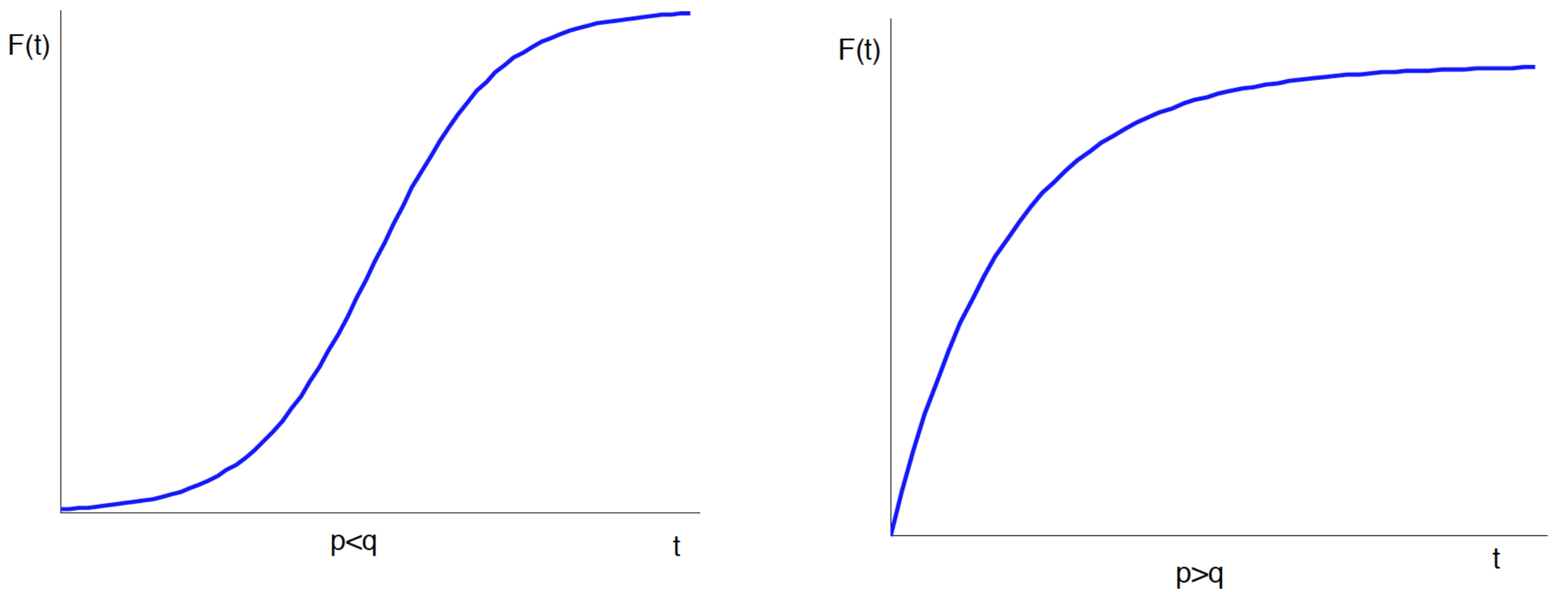
The Bass model is a special case of the Gamma/shifted Gompertz distribution (G/SG).

- Use in online social networks

The rapid, recent (as of early 2007) growth in online social networks (and other virtual communities) has led to an increased use of the Bass diffusion model. The Bass diffusion model is used to estimate the size and growth rate of these social networks.

Figure: Diffusion curves
left is for p < q and right is for p > q

- Many empirical studies have found diffusion patterns that are S-shaped (e.g. adoption of hybrid corn seeds among Iowa farmers).
- Let us interpret this in the context of adoption of new technologies.
- First adopters are almost entirely those who adopt from their spontaneous innovation (when $F(t)$ is close to 0, $\dot{F}(t) = p$).
- As process progresses, there are more agents to be imitated leading to an increase in the rate of diffusion, which eventually slows down since there are fewer agents to do the imitating.



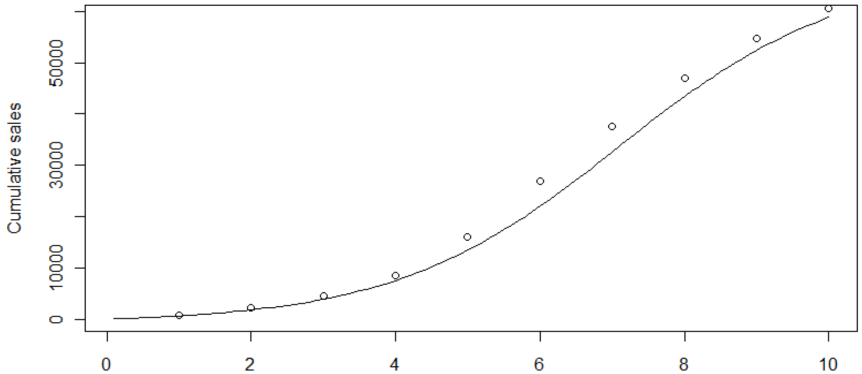
R code

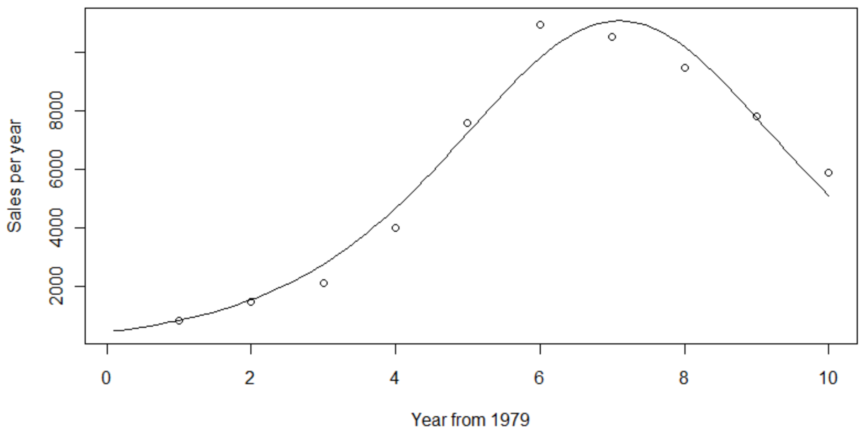
- # example
- T79 <- 1:10
- Tdelt <- (1:100) / 10
- Sales <- c(840,1470,2110,4000, 7590, 10950, 10530, 9470, 7790, 5890)
- Cusales <- cumsum(Sales)
- Bass.nls <- nls(Sales ~ M * (((P+Q)^2 / P) * exp(-(P+Q) * T79)) /(1+(Q/P)*exp(-(P+Q)*T79))^2,
-     start = list(M=60630, P=0.03, Q=0.38))
- summary(Bass.nls)

- # get coefficient
- Bcoef <- coef(Bass.nls)
- m <- Bcoef[1]
- p <- Bcoef[2]
- q <- Bcoef[3]
- # setting the starting value for M to the recorded total sales.
- ngete <- exp(-(p+q) * Tdelt)

- # plot pdf
- Bpdf <- m * ((p+q)^2 / p) * ngete / (1 + (q/p) * ngete)^2
- plot(Tdelt, Bpdf, xlab = "Year from 1979",ylab = "Sales per year", type='l')
- points(T79, Sales)

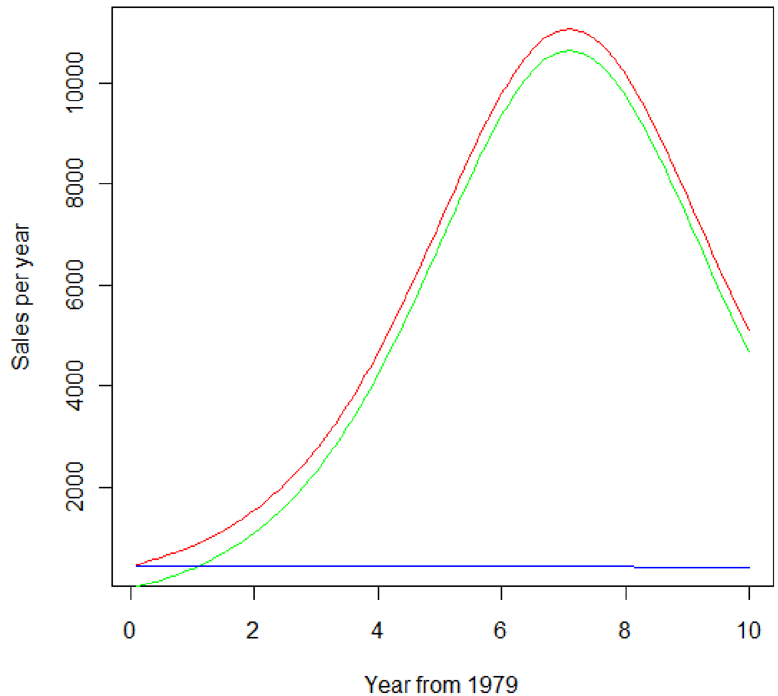
- # plot cdf
- Bcdf <- m * (1 - ngete)/(1 + (q/p)*ngete)
- plot(Tdelt, Bcdf, xlab = "Year from 1979",ylab = "Cumulative sales", type='l')
- points(T79, Cusales)





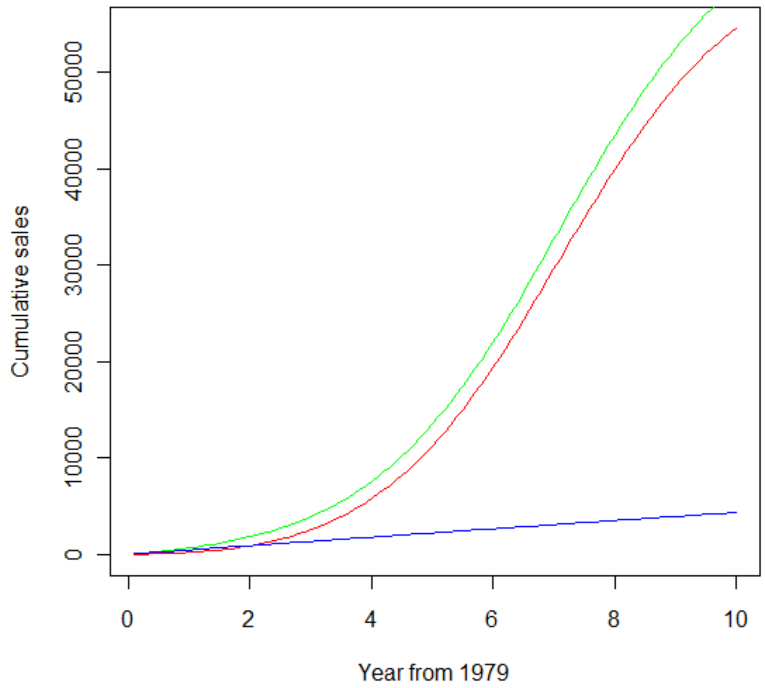
New adopters per year

- # when q=0, only Innovator without immitators.
- Ipdf<- m * ((p+0)^2 / p) * exp(-(p+0) * Tdelt) / (1 + (0/p) * exp(-(p+0) * Tdelt))^2
- Impdf<-Bpdf-Ipdf
- plot(Tdelt, Bpdf, xlab = "Year from 1979",ylab = "Sales per year", type='l', col="red")
- lines(Tdelt,Impdf,col="green")
- lines(Tdelt,Ipdf,col="blue")



Cumulative New adopters

- # when q=0, only Innovator without immitators.
- Icdf<-m * (1 - exp(-(p+0) * Tdelt))/(1 + (0/p)*exp(-(p+0) * Tdelt))
- Imcdf<-m * (1 - ngete)/(1 + (q/p)*ngete)-Icdf
- plot(Tdelt, Imcdf, xlab = "Year from 1979",ylab = "Cumulative sales", type='l', col="red")
- lines(Tdelt,Bcdf,col="green")
- lines(Tdelt,Icdf,col="blue")



What the Hell! It's close enough.

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